

Property Attribution and the Projection Postulate in Relativistic Quantum Theory

J. Finkelstein¹
Department of Physics
San José State University
San José, CA 95192, U.S.A

Abstract

State-vectors resulting from collapse along the forward light cone from a measurement interaction can be used for the attribution of both local and non-local properties.

¹Participating Guest, Lawrence Berkeley National Laboratory
e-mail: JLFINKELSTEIN@lbl.gov

There has recently (see refs. [1] - [5]) been renewed interest in the question (discussed a while ago in refs. [6] - [10]) of how, in a relativistic theory, a quantum state-vector should be assumed to collapse when a measurement is performed. Of course, even in a non-relativistic theory the postulate of state-vector collapse upon measurement is, at best, controversial; see, for example, ref. [11]. In this note, I do not wish to enter into that controversy; since I wish to explore the *additional* burden imposed on the projection postulate by the lack of a Lorentz-invariant concept of simultaneity, I will in this note naively accept that postulate, at least in a non-relativistic context. Furthermore, I will for the purpose of this note accept as correct the standard quantum expression for the probabilities of results of any measurement (which does not require any assumption of state-vector collapse; see ref. [12]). Thus I will not be considering possible modifications of that standard expression, as was suggested for example in refs. [13] and [14].

In this note I will, first, discuss the question of whether, and how, an assumption of state-vector collapse in a relativistic theory can be made which is compatible with the standard (“correct”) expression. I will argue that this can be done in more than one way, and point out that both the assumption of collapse along the backward light cone of the measurement point [7] and the assumption of collapse along the forward light cone [15, 16] can be considered as special cases of the suggestion [10] that a state-vector can be associated with *any* space-like surface. Second, I will discuss the suggestion [1, 17] that state-vectors associated with particular surfaces be used for property attribution. I will present an example in which this suggestion leads to a rather curious combination of property attributions, and propose a slight modification of this suggestion for which this curious feature does not occur. Finally, I will comment on some recently-reported experimental results [5, 14] for which the preceding considerations are relevant.

Hellwig and Kraus (ref. [7], hereafter called HK) suggested that a measurement at a space-time point M should cause the state-vector to collapse along the backward light-cone of point M ; I will refer to this suggestion as “the HK prescription”. This suggestion was criticized by Aharonov and Albert [8, 9]; this criticism was elaborated upon by Breuer and Petruccione [3] and by Ghirardi [2]. It is clear that the HK prescription does reproduce the standard [12] answer for a measurement of any local observable; however, it can be seen that this prescription would fail if it were possible to measure arbitrary non-local observables. One might attempt to defend the HK prescription on the basis of the result claimed by Landau and Peierls [18] that no non-local observable can be measured. However, Aharonov and Albert [8] showed that it *is* possible to measure *certain* non-local observables (in

a sense to be discussed below). While this result invalidates the defense of HK based on the Landau-Peierls claim, it does not yet settle the question of the viability of the HK prescription, unless it is shown that this prescription fails for those special non-local observables which in fact can be measured.

Ref. [3] states “Hellwig and Kraus have shown their prescription . . . leads to the correct predictions for all probabilities of *local* measurements. However, this state vector reduction prescription fails for the measurement of nonlocal observables”. From this statement, one might get the impression that the HK prescription can give *incorrect* predictions (or at least fails to give correct predictions) for measurements of non-local observables. In this note I will not be concerned with any ontological implications that anyone may or may not wish to draw from the HK prescription, (or from any other prescription) but I do wish to discuss whether the HK prescription does fail to give correct calculated values. I begin by summarizing the non-local measurement originally devised by Aharonov and Albert [8] in the streamlined version recently given by Ghirardi [2], and will then see whether this measurement can be correctly described by the HK prescription. Two particles, which I will call A and B and which each have isotopic spin $I = \frac{1}{2}$, are produced at a common point and then allowed to separate. The world-lines of these particles are indicated, in some particular reference frame, in fig. 1; particle A moves to the left, and particle B moves to the right. I will let $|\Psi\rangle_{AB}$ denote the initial state of these two particles.

It is desired to measure the value of the total isospin of this two-particle system. For this purpose, six “probe particles” are produced in a particular initial state (given in eq. 3.13 of [2]) which I will denote by $|\Phi\rangle_1$. Three of the probe particles interact with A at the space-time point labelled $L1$ in fig. 1, the other three interact with B at the point $R1$. These interactions are described by a unitary operator I will denote by U_1 ; all of the details are carefully specified in ref. [2], but for the present discussion I need only note a few properties of the specified interaction. If $|\Psi\rangle_{AB}$ is the isosinglet state, then U_1 acts trivially, that is

$$U_1|I = 0, I_z = 0\rangle_{AB} \otimes |\Phi\rangle_1 = |I = 0, I_z = 0\rangle_{AB} \otimes |\Phi\rangle_1. \quad (1)$$

If $|\Psi\rangle_{AB}$ is any of the isotriplet states, then the action of U (given explicitly in eqs. 3.16–3.18 of [2]) can be written

$$U|I = 1, I_z\rangle_{AB} \otimes |\Phi\rangle_1 = \sum_{I'_z=-1}^{+1} C_{I_z, I'_z} |I = 1, I'_z\rangle_{AB} \otimes |\hat{\Phi}_{I_z, I'_z}\rangle_1, \quad (2)$$

where the C_{I_z, I'_z} are numerical coefficients and where each of the states $|\hat{\Phi}_{I_z, I'_z}\rangle_1$ is orthogonal to $|\Phi\rangle_1$.

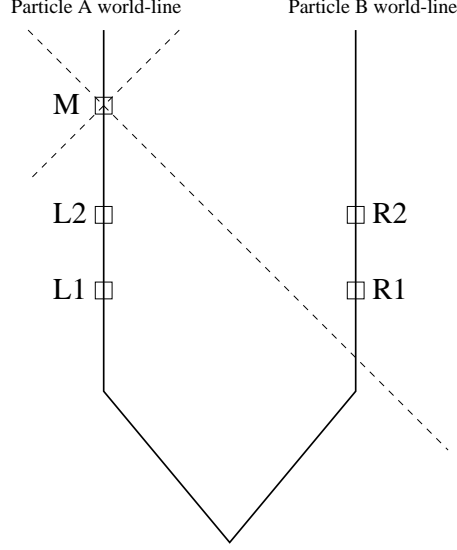


Figure 1: “Non-local measurement” of total isospin at space-time points $L1$ and $R1$, and again at $L2$ and $R2$, and local measurement of I_z of A at M . Dotted lines represent the light-cone with vertex at M .

It is important that the state $|\Phi\rangle_1$ can be distinguished from any of the states $|\hat{\Phi}_{I_z, I'_z}\rangle_1$ by subsequent measurements done on the individual probe particles. This makes it possible to say, assuming that these subsequent measurements are in fact performed, that the state $|\Phi\rangle_1$ is (or else is not) found. It then follows, from eqs. 1 and 2 that

- α If the initial state is the isosinglet $|I = 0, I_z = 0\rangle_{AB}$, then $|\Phi\rangle_1$ will surely be found.
- β Now assume that a second, similar, measurement is also performed, with six new probe particles in an initial state $|\Phi\rangle_2$ (analogous to $|\Phi\rangle_1$) at the points labelled $L2$ and $R2$ in fig. 1. Then, for arbitrary initial state of the AB system, if $|\Phi\rangle_1$ is in fact found, then with probability one $|\Phi\rangle_2$ will also be found.

These two features, especially feature β , suggest that we may regard the observation of $|\Phi\rangle_1$ as a preparation of the AB system in the isosinglet state, and the subsequent observation of $|\Phi\rangle_2$ as a verification of that state. Although this procedure may *result* in a measurement (at least in the sense of features α and β) of the non-local observable I^2 , it is important to realize that it *consists* of purely-local interactions and measurements.

Now suppose that, at the point labelled M in fig. 1, a measurement of I_z of particle A is performed. Clearly this does not change the preceding discussion in any way, and in particular features α and β are still present. Let U_2 denote the unitary operator for the interactions at $L2$ and $R2$ (with properties analogous to U_1), and let P_M be the projection operator onto whichever state of A is in fact found in the measurement at M . Then after the interactions at $L1$, $L2$, $R1$, and $R2$, and this measurement at M , the joint state of A , B , and all of the probe particles would be given by

$$|\text{final}\rangle = P_M U_2 U_1 |\Psi\rangle_{AB} \otimes |\Phi\rangle_1 \otimes |\Phi\rangle_2. \quad (3)$$

Features α and β of course follow from the expression for $|\text{final}\rangle$ given in eq. 3 (the presence of P_M being irrelevant); feature β follows from the fact that the projection operator $\mathbf{I}_A \otimes \mathbf{I}_B \otimes |\Phi\rangle_{11} \langle \Phi| \otimes (\mathbf{I}_2 - |\Phi\rangle_{22} \langle \Phi|)$ annihilates $|\text{final}\rangle$. (Note that the symbol \mathbf{I} denotes an identity, not an isospin, operator.)

Features α and β are features of the experimental results which, according to standard quantum theory, would be obtained if an experiment of the type discussed were actually performed. To say that an observation of $|\Phi\rangle_1$ prepares the AB system in the isoscalar state is to make an interpretation of those features. The HK prescription may lead to a different interpretation but, as will be shown below, to the same experimental features. So let me consider this same example — interactions at $L1$, $L2$, $R1$, and $R2$, and the measurement at M — as described by the HK prescription. I first consider the case, illustrated in fig. 1, in which the points $R1$ and $R2$ both have space-like separation from the measurement point M ; this means, in the HK prescription, that collapse due to that measurement has already occurred at the points $R1$ and $R2$. But that collapse has not occurred at $L1$ or $L2$, and so there is no single state-vector which describes the system at $L1$ and at $R1$ (nor one for the system at $L2$ and $R2$). Nevertheless, HK could proceed in the following way: The unitary operator U_1 can (necessarily, since the interactions are local) be written

$$U_1 = (\mathbf{I}_L \otimes U_{R1})(U_{L1} \otimes \mathbf{I}_R), \quad (4)$$

where U_{R1} acts only on the particles present at $R1$, and U_{L1} only on the particles present at $L1$. Similarly, $U_2 = (\mathbf{I}_L \otimes U_{R2})(U_{L2} \otimes \mathbf{I}_R)$. The effect of the measurement at M is to apply the projection operator P_M to the state vector, and to use the collapsed state at $R1$ and $R2$ requires that P_M be applied *before* the operators U_{R1} or U_{R2} . So HK would calculate

$$|\text{final}\rangle = (\mathbf{I}_L \otimes U_{R2})(\mathbf{I}_L \otimes U_{R1})P_M(U_{L2} \otimes \mathbf{I}_R)(U_{L1} \otimes \mathbf{I}_R)|\Psi\rangle_{AB} \otimes |\Phi\rangle_1 \otimes |\Phi\rangle_2. \quad (5)$$

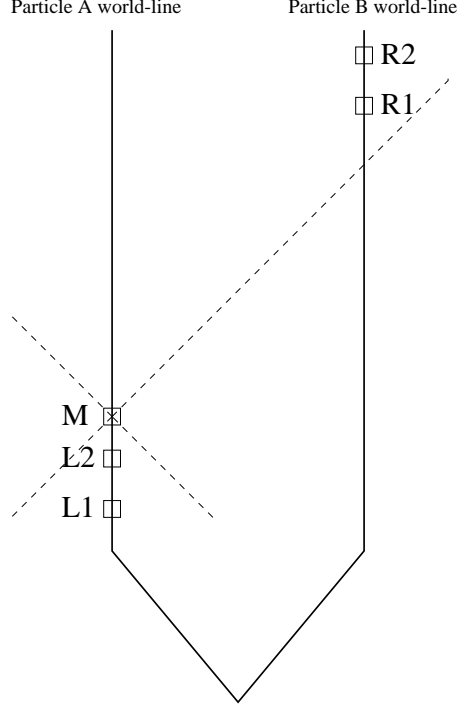


Figure 2: The same as fig. 1, except that here $R1$ and $R2$ are unambiguously in the future of the measurement at M .

However, since P_M commutes with U_{R1} and with U_{R2} , and U_{R1} commutes with U_{L2} , the left-hand sides of eqs. 3 and 5 are equal. Since, as noted above, features α and β are consequences of the form of $|\text{final}\rangle$, we see that these features are valid for HK also.

Thus although the HK prescription does not assign a unique state-vector to the AB system at points $L2$ and $R2$, it does give correct calculated values. Perhaps this point could be clarified by a slight change in the example. Suppose that, as indicated in fig. 2, the points $R1$ and $R2$ were unambiguously in the *future* of the measurement point M , all other aspects of this example being unchanged. Then one could still calculate $|\text{final}\rangle$ as in eq. 5, and would see that features α and β were valid for this changed example also. However, in this case not everybody would feel compelled to interpret these features as implying that the AB system at points $L2$ and $R2$ must be described by a unique state-vector.

Of course one could construct more elaborate examples than the one considered here, perhaps with more than a single measurement-induced col-

lapse. Any such example could be analyzed, using the HK states, by the following procedure: First, pick any single world-line (it might be convenient, but it is not necessary, to pick the world-line of part of the quantum system being described), and second, to use the HK states defined along that world-line, which means to apply the projection operators which collapse the state-vector in the order in which the backward light cones of the measurements which produced those collapses intersect the chosen world-line. The HK prescription is constructed to give results in complete agreement with standard quantum theory for any collection of local measurements, and that remains true whether or not those measurements have the *effect* (as in the example considered above) of measuring a non-local observable.

The HK prescription assigns to each space-time point a single state-vector, and as I have argued above these state-vectors suffice to reproduce the standard quantum result. However, other prescriptions are also possible. My personal preference (expressed in ref. [15]; see also Mosley [16]) is for the assumption that collapse occurs along the *forward* light cone of the measurement point. In this prescription, projection operators are applied in the order in which the forward light cones of the measurements intersect the chosen world-line.

Aharonov and Albert [10] have shown how a state-vector can be assigned to each (unbounded) space-like surface. Any such surface separates the remainder of space-time into two disjoint regions, which may be called the regions “before” and “after” that surface (a point M is “before” (“after”) the surface if the forward (backward) light cone with vertex at M intersects that surface); Aharonov and Albert assign to each space-like surface the state-vector which has experienced collapses due to all those measurement interactions which are “before” the surface. Suppose we accept this assignment, and suppose that, for any space-time point P we let $\eta(P)$ denote the forward light cone with vertex at P . We can then simply *define*, for each point P , the state $|\Psi(\eta(P))\rangle$ to be the state assigned in ref. [10] to the surface $\eta(P)$ (here I ignore, as do refs. [1, 17] in a similar context discussed below, the fact that a light cone is not quite a space-like surface). This definition, made entirely within the scheme of ref. [10], certainly does assign to each point P a unique state-vector. Furthermore, since for any space-time point M the condition “ M is before $\eta(P)$ ” is equivalent to the condition “ P is after the backward light cone from M ”, it is not hard to see that $|\Psi(\eta(P))\rangle$ is exactly the state which HK assign to the point P . This indicates that the HK prescription, rather than being in conflict with the prescription of Aharonov and Albert, is really a special case of the latter.

Similarly, if we let $\sigma(P)$ denote the backward light cone with vertex at P ,

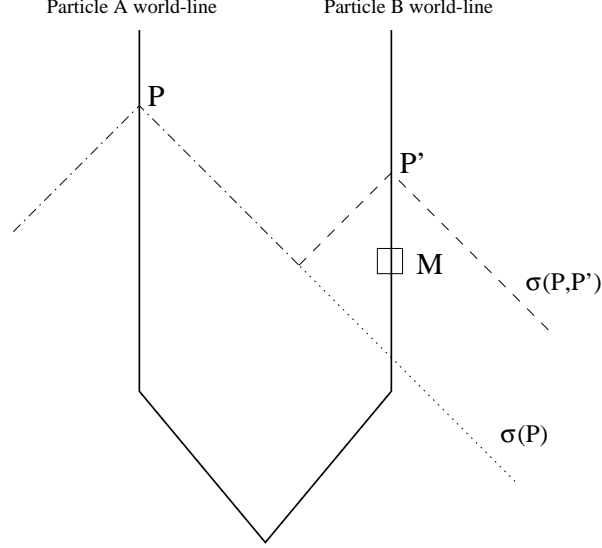


Figure 3: Surfaces used for property attributions in ref. [1, 17]. Dashed lines represent $\sigma(P, P')$, dotted line $\sigma(P)$, and dashed-dotted lines both surfaces coinciding.

we can define $|\Psi(\sigma(P))\rangle$ to be the state which Aharonov and Albert assign to the surface $\sigma(P)$. It can then be seen that $|\Psi(\sigma(P))\rangle$ is the same as the state assigned to P by the prescription (refs. [15, 16]) that collapse occurs along the *forward* light cone of the measurement. So this prescription also can be considered as a special case of the prescription of ref. [10].

Ghirardi ([1, 17]) has suggested that this state-vector $|\Psi(\sigma(P))\rangle$ should be used for the attribution of local properties to a quantum system at point P (for simplicity, I ignore the fact that Ghirardi really modifies the surface $\sigma(P)$ by the surface on which initial conditions are specified). That is, he proposes that a system at the space-time point P should be said to possess a definite value of a local quantity just in case that $|\Psi(\sigma(P))\rangle$ is an eigenvector of the operator associated with that quantity (for brevity, let's read that "local properties of a system at P are given by $|\Psi(\sigma(P))\rangle$ "). For the attribution of non-local properties (such as the total isospin of a two-particle system) at points P and P' , he defines a surface $\sigma(P, P')$ as illustrated in fig. 3 (this is the surface which lies immediately "after" the union of $\sigma(P)$ with $\sigma(P')$). Defining $|\Psi(\sigma(P, P'))\rangle$ to be the state associated with the surface $\sigma(P, P')$, he proposes that non-local properties be given by $|\Psi(\sigma(P, P'))\rangle$.

This use of different surfaces for local and for non-local properties can lead to some curious attributions, as is shown by the following example. Suppose that the initial state of a pair of particles called A and B is a superposition of a $\pi\pi$ and a $K\bar{K}$ pair. Since a π has $I = 1$ and a K or \bar{K} has $I = \frac{1}{2}$, I can write

$$\begin{aligned} |K\bar{K}, I = 0\rangle &= \frac{1}{\sqrt{2}}(|K^+\rangle_A |K^-\rangle_B - |K^0\rangle_A |\bar{K}^0\rangle_B) \\ |\pi\pi, I = 2, I_z = 0\rangle &= \frac{1}{\sqrt{6}}|\pi^+\rangle_A |\pi^-\rangle_B + \sqrt{\frac{2}{3}}|\pi^0\rangle_A |\pi^0\rangle_B + \frac{1}{\sqrt{6}}|\pi^-\rangle_A |\pi^+\rangle_B. \end{aligned}$$

Now let me take the initial state of the AB system to be

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|K\bar{K}, I = 0\rangle + |\pi\pi, I = 2, I_z = 0\rangle), \quad (6)$$

and suppose that at the position marked M in fig. 3 the particle type but not the charge is measured (i.e., the hypercharge of particle B is measured), and particle B is found to be a π . Then since M is “before” $\sigma(P, P')$ but “after” $\sigma(P)$ we would have

$$|\Psi(\sigma(P, P'))\rangle = |\pi\pi, I = 2, I_z = 0\rangle. \quad (7)$$

but $|\Psi(\sigma(P))\rangle$ is the same as the initial state $|\Psi\rangle_{AB}$. Since $|\Psi(\sigma(P, P'))\rangle$ is an eigenstate of I^2 , the proposed [1, 17] property attribution would be that the combined state of A at P and B at P' definitely has $I = 2$, although the particle type of particle A (which being a local property is determined by $|\Psi(\sigma(P))\rangle$) is not definitely either π or K . What is curious about this is that it can be seen from the initial state that if $I = 2$ then particle A is definitely π . In fact, independently of the chosen initial state, a two-meson system with $I = 2$ can never contain a K .

This curious situation would not occur if both local and non-local properties at P were all given by $|\Psi(\sigma(P))\rangle$; in this example, that would mean that at point P the value of I would *not* be definite. The property attribution which is suggested by the prescription given in [15, 16] is that, *as seen from point P*, all properties are given by $|\Psi(\sigma(P))\rangle$, whether or not (any part of) the system being described happens to be located at P . For local properties of a system located at P , this attribution agrees with the one suggested in [1, 17].

Finally, let me comment on two recent experimental results for which the above considerations are relevant. Scarani, Tittel, Zbinden and Gisin [5] report on an experimental determination of a lower bound of $(1.5 \times 10^4)c$ for what they call the “speed of quantum information” in the frame of the cosmic microwave background radiation. What is actually determined,

as can be seen from eq. 1 of [5], is the distance between measurements performed on a pair of entangled photons divided by the time difference of those measurements. It is not clear that this represents the speed of anything, since it is not clear that there is anything which propagates from one measurement to the other (e.g., according to the prescription of refs. [15] and [16], not even state-vector collapse exceeds c). In fact, footnote 14 of [5] suggests that, in spite of the title of that paper, the “speed of quantum information” might not be the most appropriate term for the quantity which that experiment determined; I would agree with that suggestion.

Zbinden, Brendel, Gisin and Tittel [14] report on an attempt to find deviations from standard quantum predictions in a case in which the detectors for two entangled photons are in relative motion in such a way that each detector acts first in its own reference frame. No such deviations were found. As discussed above, the HK prescription for state-vector collapse is completely consistent with standard quantum predictions (as are the prescriptions of ref. [10] and of refs. [15, 16]). Furthermore, the CSL model (reviewed in refs. [1, 2]) would presumably give results which would be indistinguishable from the standard predictions to this level of precision. This casts doubt on the assertion made in the last sentence of [14] that experimental results which agree with standard quantum theory “make it more difficult to view the ‘projection postulate’ as a compact description of a real physical phenomenon”.

Acknowledgements

I acknowledge the hospitality of the Lawrence Berkeley National Laboratory.

References

- [1] G. C. Ghirardi, in *Open Systems and Measurement in Relativistic Quantum Theory*, edited by H. P. Breuer and F. Petruccione, Springer, Berlin, p. 117 (1999).
- [2] G. C. Ghirardi, “Local measurements of nonlocal observables and the relativistic reduction process”, preprint quant-ph/0003149 (2000).
- [3] H. P. Breuer and F. Petruccione, in *Open Systems and Measurement in Relativistic Quantum Theory*, edited by H. P. Breuer and F. Petruccione, Springer, Berlin, p. 1 (1999).

- [4] R. A. Mould, “A defense of Hellwig-Kraus reductions”, preprint quant-ph/9912044 (1999).
- [5] V. Scarani, W. Tittel, H. Zbinden, and N. Gisin, “The speed of quantum information and the preferred frame: analysis of experimental data”, preprint quant-ph/0007008 (2000).
- [6] I. Block, Phys. Rev. **156**, 1377 (1967).
- [7] K.-E. Hellwig and K. Kraus, Phys. Rev. D **1**, 566 (1970).
- [8] Y. Aharonov and D. Z. Albert, Phys. Rev. D **21**, 3316 (1980).
- [9] Y. Aharonov and D. Z. Albert, Phys. Rev. D **24**, 359 (1981).
- [10] Y. Aharonov and D. Z. Albert, Phys. Rev. D **29**, 228 (1984).
- [11] J. S. Bell, in *Sixty-two years of uncertainty*, edited by A. I. Miller, Plenum, New York, p. 17 (1990).
- [12] E. P. Wigner, Am. J. Phys. **31**, 6 (1963).
- [13] A. Suarez and V. Scarani, Phys. Lett. A **232**, 9 (1997).
- [14] H. Zbinden, J. Brendel, N. Gisin, and W. Tittel, “Experimental test of non-local quantum correlation in relativistic configurations”, preprint quant-ph/0007009 (2000).
- [15] J. Finkelstein, Found. Phys. Lett. **5**, 383 (1992); Phys. Lett. A **188**, 117 (1994).
- [16] S. N. Mosley, Phys. Lett. A **182**, 1 (1993).
- [17] G. C. Ghirardi, Found. Phys. Lett. **9**, 313 (1996).
- [18] L. Landau and R. Peierls, Z. Phys. **69**, 56 (1931).